

UNIT-3: FILTERS

FILTER:

A filter is a circuit capable of passing certain frequencies while attenuating other frequencies.

Practical applications of filter:

- 1- Radio communications: filters enable radio receivers to only receive the desired signals.
- 2- DC Power Supplies: filters eliminate undesired high frequencies (i.e., noise) that are present on AC input lines. Also, filters are used on power supply output to reduce ripples.
- 3- Audio electronics: Network of filters is used to channel low-frequency audio to woofers, etc.
- 4- Analog-to-Digital Conversion: filters are placed in front of an ADC input to reduce aliasing.

[ALIASING: It is an effect that causes different signals to become indistinguishable when sampled for A-to-D conversion.]

TYPES OF FILTERS

The four major types of filters include -

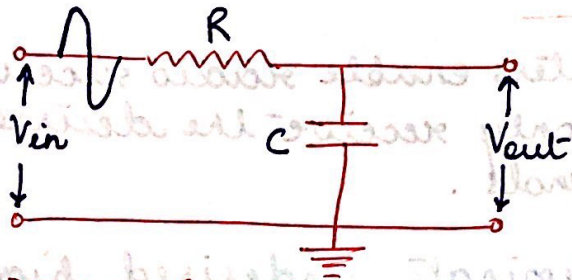
- 1- Low Pass Filter (LPF)
- 2- High Pass Filter (HPF)
- 3- Band Pass Filter (BPF)
- 4- Band Stop Filter (Band Reject or Notch Filter)

Filters can also be placed in one of the two categories:

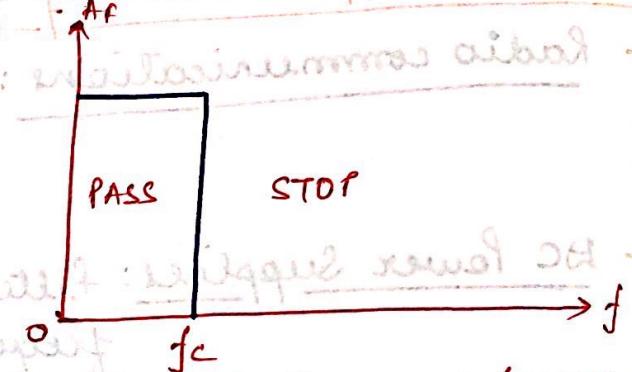
- 1- Passive filters (include only resistors, capacitors and inductors)
- 2- Active filters (use active components such as op-amps in addition to resistors and capacitors but not inductors)

① PASSIVE LOW PASS FILTER

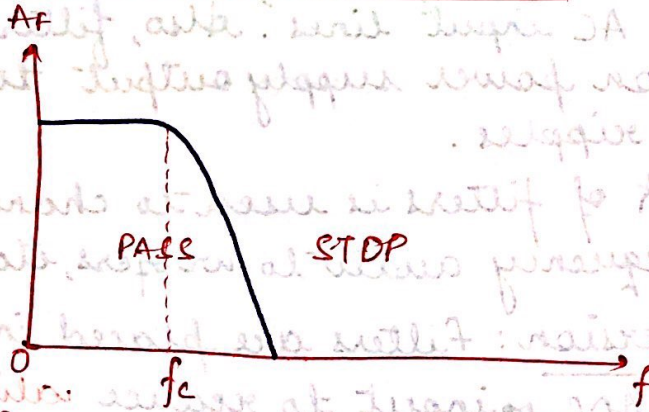
- * It allows low frequency signals from 0 Hz to its cut-off frequency, f_c to pass while blocking the higher frequencies.
- * It is also called "first-order filter" or "one-pole filter" because it has only "one" reactive component (capacitor) in the circuit.



RC Low Pass Filter Circuit



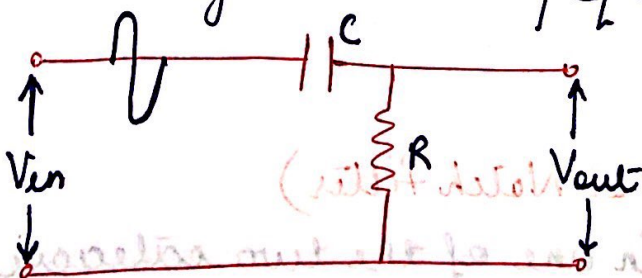
Ideal LPF Response Curve



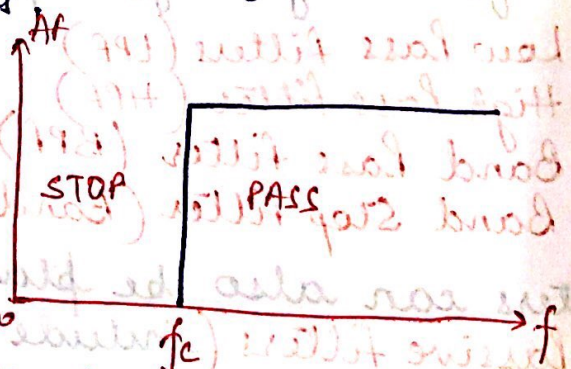
Practical LPF Response Curve

② PASSIVE HIGH PASS FILTER

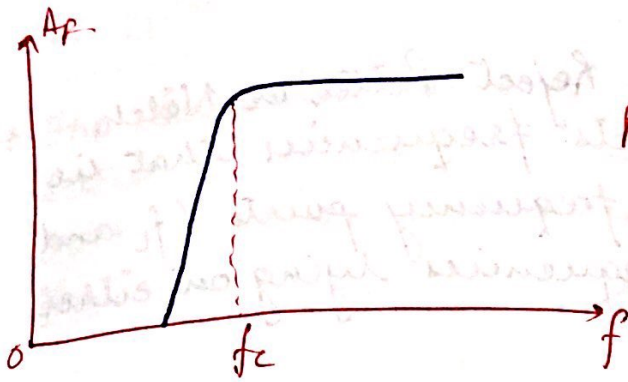
It only allows high frequency signals from its cut off frequency, f_c to infinity to pass through while blocking the lower frequencies.



High Pass Filter Circuit



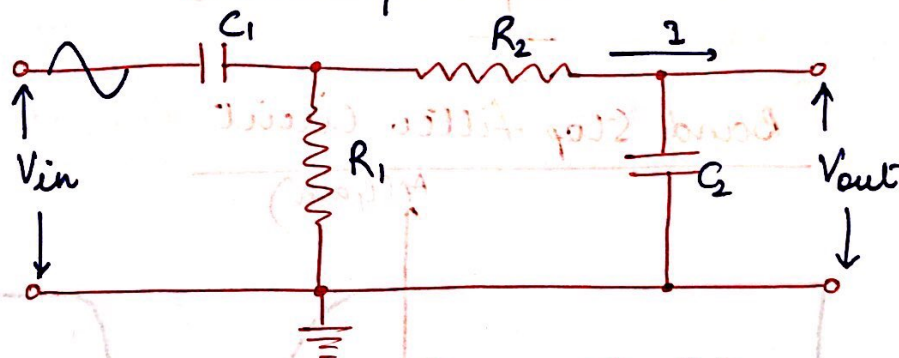
Ideal HPF Response Curve



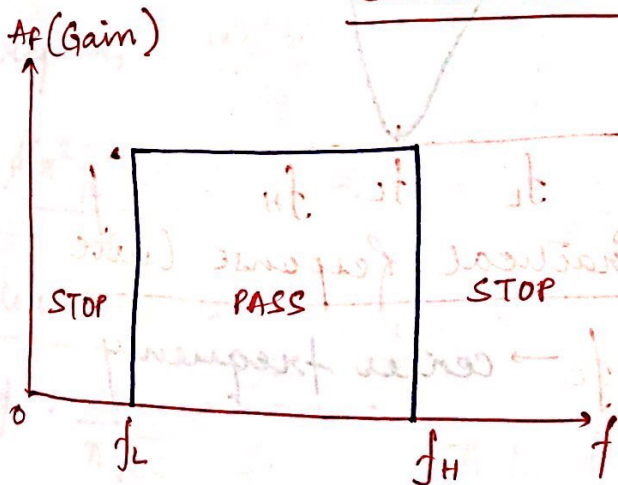
Practical HPF Response Curve.

(3) Band Pass Filter:

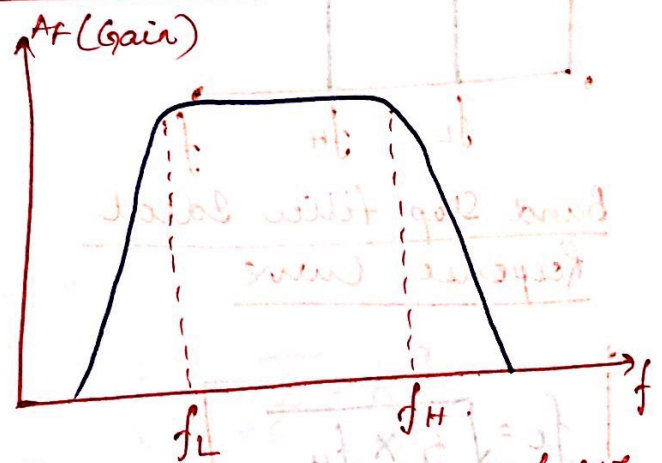
It passes signals or frequencies within a certain "band" without distorting the input signal or introducing extra noise. This band of frequencies can be any width and is called filter bandwidth.



Band Pass filter circuit



Ideal BPF Response Curve

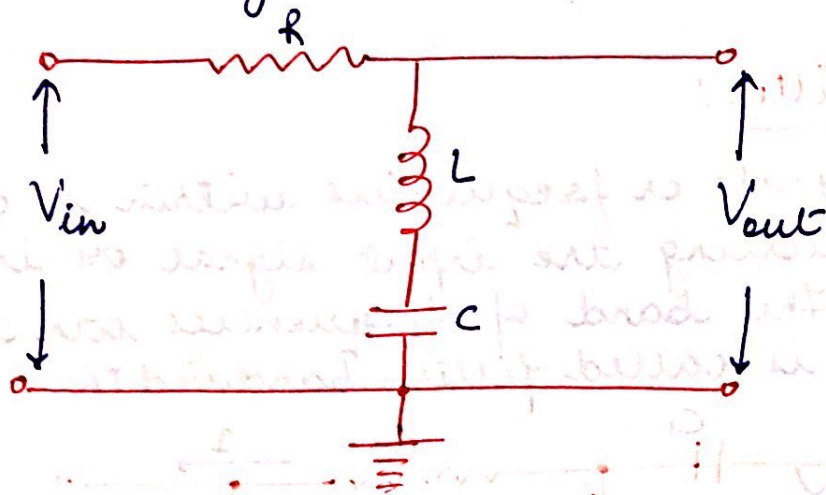


Practical BPF Response Curve

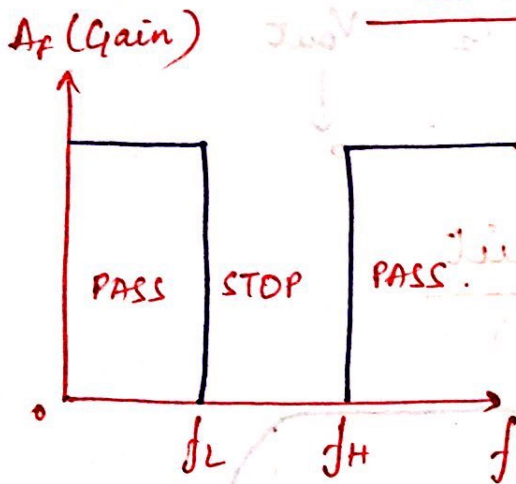
f_L → lower cut-off frequency
 f_H → higher cut-off frequency.

(4) Band Stop Filter:

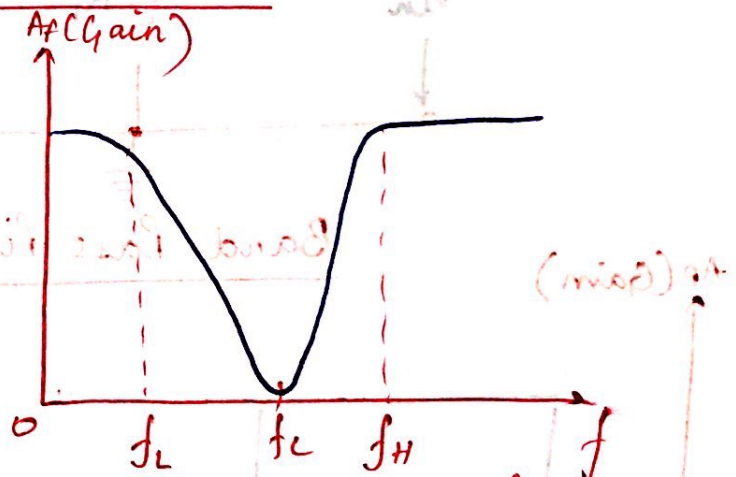
It is also known as Band Reject filter or Notch filter. It blocks and rejects frequencies that lie between its two cut-off frequency points (f_L and f_H) and passes all the frequencies lying on either side of this range.



Band Stop filter circuit



Band Stop filter Ideal Response Curve



Practical response curve

$f_c \rightarrow$ center frequency

$$f_c = \sqrt{f_L \times f_H}$$

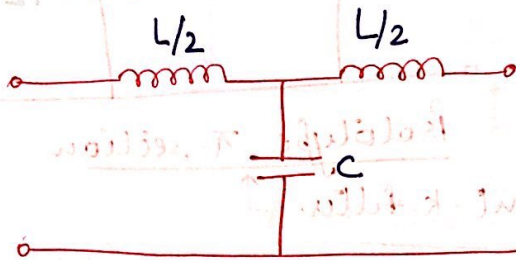
PROTOTYPE FILTER SECTION OR CONSTANT K-FILTERS

- * A constant k-filter is a T or π -network in which series or shunt impedance Z_1 and Z_2 are connected by the following relationship:

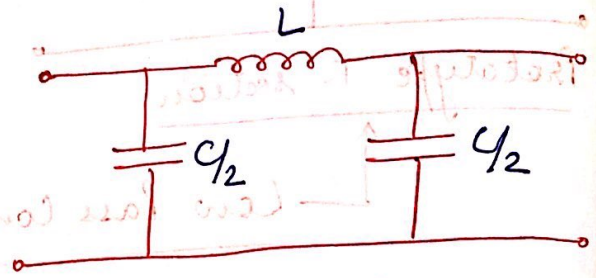
$$Z_1 Z_2 = k^2$$

$k \rightarrow$ real constant called nominal or design impedance of the constant-k filter.

Low Pass Constant k-filter



Prototype low pass T-section



Prototype low pass π -section

- * The building block of constant-k filter is the half section L-network.

In the above circuit diagrams,

$$Z_1 = j\omega L, \quad Z_2 = -j/\omega C$$

$$\text{Hence, } Z_1 Z_2 = j\omega L \times \frac{-j}{\omega C} = \frac{L}{C} \quad [\because j \times j = -1]$$

$$\text{Thus, } Z_1 Z_2 = k^2 = \frac{L}{C} \Rightarrow k = \sqrt{\frac{L}{C}}$$

$\therefore$ Characteristic impedance, $Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$

Substituting the values of Z_1 and Z_2 , we get -

$$Z_0 = \sqrt{\frac{(j\omega L)^2}{4} + \frac{L}{C}} = \sqrt{\frac{-\omega^2 L^2}{4} + \frac{L}{C}} = \sqrt{\frac{L}{C}} \sqrt{1 - \frac{\omega^2 LC}{4}}$$

Z_0 is real if $\frac{\omega^2 LC}{4} < 1$ and imaginary if $\frac{\omega^2 LC}{4} > 1$

Since, the cut-off frequency f_c is the frequency at which Z_0 changes from real to imaginary.

Therefore, $\frac{\omega_c^2 LC}{4} = 1$

$\Rightarrow \frac{4\pi^2 f_c^2 LC}{4} = 1 \Rightarrow f_c = \frac{1}{\pi\sqrt{LC}}$

Calculating the values of L and C

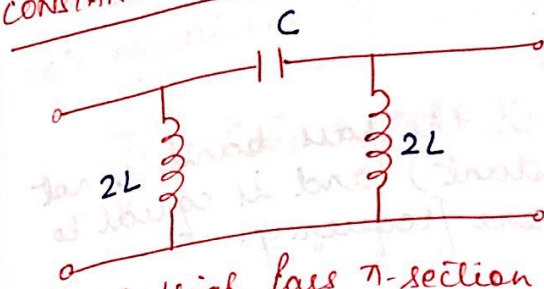
$f_c = \frac{1}{\pi\sqrt{LC}} \times \frac{\sqrt{L}}{\sqrt{L}} = \frac{1}{\pi L} \sqrt{\frac{L}{C}} \quad \left[\because \sqrt{\frac{L}{C}} = k \right]$

$\therefore f_c = \frac{k}{\pi L} \Rightarrow \boxed{L = \frac{k}{\pi f_c}}$

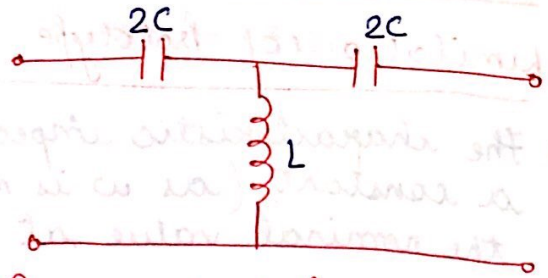
$f_c = \frac{1}{\pi\sqrt{LC}} \times \frac{\sqrt{C}}{\sqrt{C}} = \frac{1}{\pi C} \sqrt{\frac{C}{L}} = \frac{1}{\pi C \times \sqrt{\frac{L}{C}}}$

$\therefore f_c = \frac{1}{\pi C \cdot k} \Rightarrow \boxed{C = \frac{1}{\pi f_c k}}$

CONSTANT-K HIGH PASS FILTERS:



Prototype High Pass π -section



Prototype High Pass T-section

Here, $Z_1 = \frac{-j}{\omega C}$ and $Z_2 = j\omega L$

$$Z_1 Z_2 = \frac{-j}{\omega C} \times j\omega L = \frac{L}{C}$$

$$\therefore Z_1 Z_2 = k^2 \Rightarrow k^2 = \frac{L}{C} \Rightarrow \boxed{k = \sqrt{\frac{L}{C}}}$$

characteristic impedance, $Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$

$$Z_0 = \sqrt{\frac{1}{4} \cdot \left(\frac{-j}{\omega C}\right)^2 + \frac{L}{C}} = \sqrt{\frac{-1}{4\omega^2 C^2} + \frac{L}{C}} = \sqrt{\frac{L}{C}} \sqrt{1 - \frac{1}{4\omega^2 LC}}$$

Z_0 is real if $\frac{1}{4\omega^2 LC} < 1$ and imaginary if $\frac{1}{4\omega^2 LC} > 1$

and cut off frequency f_c is the one at which Z_0 change from real to imaginary

Therefore, $\frac{1}{4\omega_c^2 LC} = 1 \Rightarrow \frac{1}{4 \times 4\pi^2 f_c^2 LC} = 1$

$$\therefore \boxed{f_c = \frac{1}{4\pi\sqrt{LC}}}$$

Calculating the values of L and C

$$f_c = \frac{1}{4\pi\sqrt{LC}} \times \frac{\sqrt{L}}{\sqrt{L}} = \frac{1}{4\pi L} \sqrt{\frac{L}{C}} = \frac{1}{4\pi L} \times k \left[\because k = \sqrt{\frac{L}{C}} \right]$$

$$\therefore \boxed{L = \frac{k}{4\pi f_c}}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}} \times \frac{\sqrt{C}}{\sqrt{C}} = \frac{1}{4\pi C} \sqrt{\frac{C}{L}} = \frac{1}{4\pi C} \times \frac{1}{\sqrt{L/C}} = \frac{1}{4\pi C} \times \frac{1}{k}$$

$$\therefore \boxed{C = \frac{1}{4\pi k f_c}}$$

M-DERIVED FILTERS

Limitations of Prototype filters:

- 1) The characteristic impedance in the pass band is not a constant (as ω is not constant) and is equal to the nominal value at only one frequency.
- 2) The attenuation does not take place abruptly at the cut-off frequency so there is no clear line of demarcation between the pass band and the stop band.

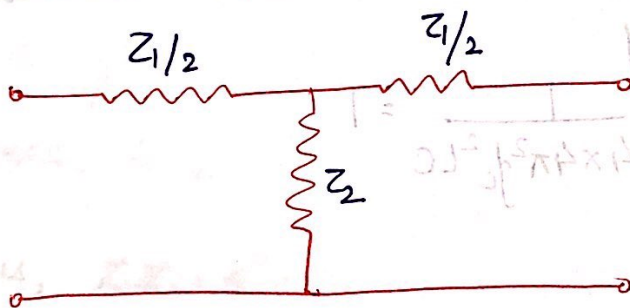
Need for m-Derived filters:

The cut-off frequency of the derived filter shall be same as that of the original filter.

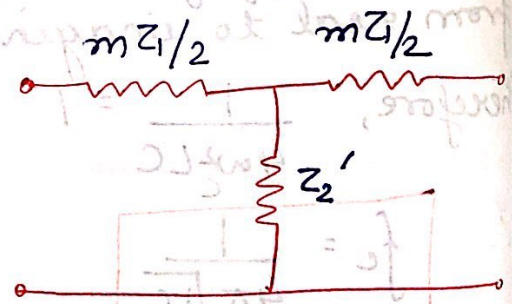
The characteristic impedance of the derived filter shall be identical with that of the original in the pass band.

The attenuation of the derived filter shall be fixed arbitrarily.

Derivation of m-Derived T-section



T-section



m-derived section

For a prototype section, the characteristic impedance is given by - $Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$ — (1)

Similarly, for a m-derived section,

$$Z_0 = \sqrt{\frac{(mZ_1)^2}{4} + (mZ_1) Z_2'} \quad \text{--- (2)}$$

To maintain same value of characteristic impedance, both equations ① and ② must be equated -

$$\frac{Z_1^2}{4} + Z_1 Z_2 = \frac{m^2 Z_1^2}{4} + m Z_1 Z_2'$$

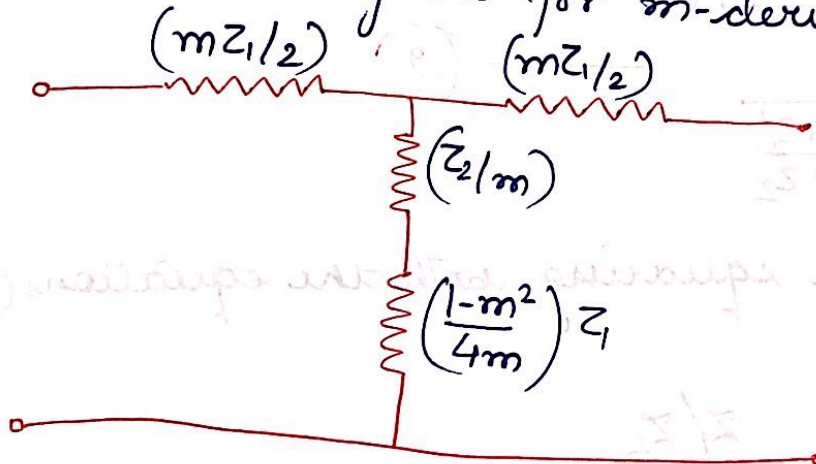
$$m Z_1 Z_2' = \frac{Z_1^2}{4} - \frac{m^2 Z_1^2}{4} + Z_1 Z_2$$

$$m Z_1 Z_2' = \frac{(1-m^2)}{4} Z_1^2 + Z_1 Z_2$$

$$Z_2' = \left(\frac{1-m^2}{4m} \right) Z_1 + \frac{Z_2}{m} \quad \text{--- ③}$$

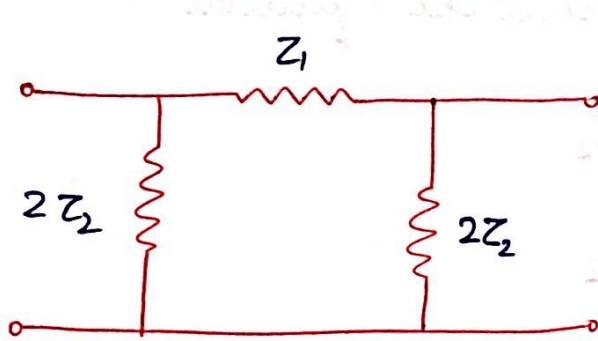
From equation ③, it is clear that shunt arm of m-derived section is a series connection of two impedances - $\left(\frac{Z_2}{m} \right)$ and $\left(\frac{1-m^2}{4m} \right) Z_1$ if $0 < m < 1$ (as Z_1 cannot be negative)

Thus, the circuit diagram for m-derived section is:

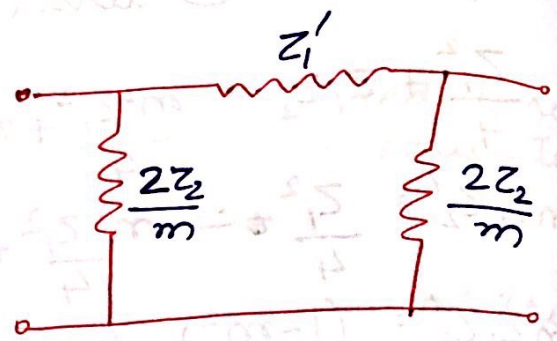


m-derived
T-section

Derivation of m-derived π -Section



(a) π -Section



(b) m-derived π -section

for a prototype π -section, Characteristic Impedance Z_0 is given by -

$$Z_0 = \sqrt{\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}}} \quad \text{--- (1)}$$

Similarly for a m-derived π -section, the characteristic impedance Z_0 is given by -

$$Z_0 = \sqrt{\frac{\frac{Z_1' Z_2}{m}}{1 + \frac{m Z_1'}{4Z_2}}} \quad \text{--- (2)}$$

Equating and squaring both the equations (1) and (2) we get -

$$\frac{Z_1 Z_2}{1 + \frac{Z_1}{4Z_2}} = \frac{\frac{Z_1' Z_2}{m}}{1 + \frac{m Z_1'}{4Z_2}}$$

$$\therefore Z_1 Z_2 \left[1 + \frac{m Z_1'}{4Z_2} \right] = \frac{Z_1' Z_2}{m} \left[1 + \frac{Z_1}{4Z_2} \right]$$

$$\therefore Z_1 Z_2 + Z_1' \left(\frac{m Z_1}{4} \right) = Z_1' \left(\frac{Z_2}{m} + \frac{Z_1}{4m} \right)$$

$$\therefore Z_1 Z_2 = Z_1' \left(\frac{Z_2}{m} + \frac{Z_1}{4m} \right) - Z_1' \left(\frac{m Z_1}{4} \right)$$

$$\therefore Z_1' \left[\frac{Z_2}{m} + \frac{Z_1}{4m} - \frac{mZ_1}{4} \right] = Z_1 Z_2$$

$$\therefore Z_1' \left[\frac{Z_2}{m} + \frac{Z_1}{4m} (1-m^2) \right] = Z_1 Z_2$$

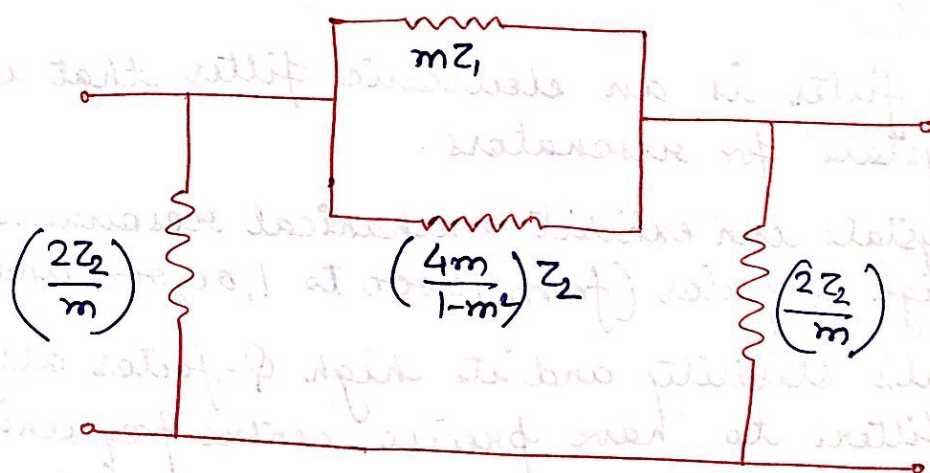
$$\therefore Z_1' = \frac{Z_1 Z_2}{\frac{Z_2}{m} + \left(\frac{1-m^2}{4m} \right) Z_1} = \frac{(mZ_1)(4Z_2)}{4Z_2 + (1-m^2)Z_1}$$

Multiplying numerator and denominator by the factor $\left(\frac{m}{1-m^2} \right)$, we get -

$$Z_1' = \frac{(mZ_1) \left(\frac{4m}{1-m^2} \right) Z_2}{\left(\frac{4m}{1-m^2} \right) Z_2 + mZ_1} \quad \text{--- (3)}$$

from equation (3), it is clear that the series arm Z_1' of m -derived section is a parallel combination of two impedances (mZ_1) and $\left(\frac{4m}{1-m^2} \right) Z_2$ with condition $0 < m < 1$ (as Z_2 can't be negative).

The m -derived π -section is as shown below:



m -derived π -section

CRYSTAL FILTERS OR PIEZOELECTRIC FILTERS

Piezoelectric Effect

- * When pressure is applied to a piezoelectric material (like quartz), it creates an electrical charge in that material. This phenomenon is referred to as piezoelectric effect.
- * Thus, we can say that the piezoelectric effect occurs through compression of a piezoelectric material.

Piezoelectric Materials:

- * These are the materials that can produce electricity due to mechanical stress, such as compression.
- * These materials can also deform when voltage is applied.
- * Some examples of piezoelectric materials are:
 - PZT (Lead Zirconate Titanate)
 - Barium Titanate
 - Lithium Niobate
 - Quartz — Natural

} Man-made material

Crystal Filters:

- * A crystal filter is an electronic filter that uses quartz crystals for resonators.
- * Quartz crystals can exhibit mechanical resonances with a very high Q-factor (from 10,000 to 1,00,000 and greater).
- * The crystal's stability and its high Q-factor allow crystal filters to have precise centre frequencies and steep band-pass characteristics.
- * Q-factor or Quality factor is a dimensionless parameter that describes how underdamped an oscillator is. It is the ratio of a resonator's centre frequency to its bandwidth.

* Crystal filters are commonly used in radio receivers, telecommunications, signal generation and GPS devices.

ACTIVE FILTERS

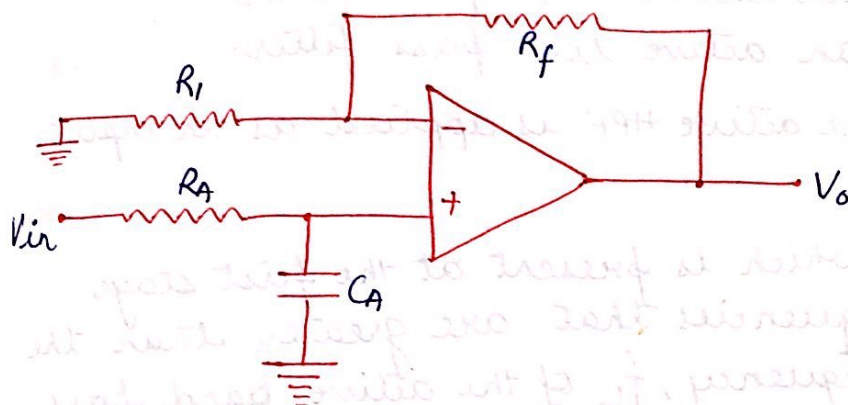
* Active filters are the electronic circuits, which consist of active element like op-amp along with passive elements like resistors and capacitors but not inductors.

* They are classified as-

- Active Low Pass filter
- Active High Pass filter
- Active Band Pass filter
- Active Band Stop filter

(1) ACTIVE LOW PASS FILTER :

If an active filter passes only low frequency components and blocks or rejects all other high frequency components then it is called as an active low pass filter.

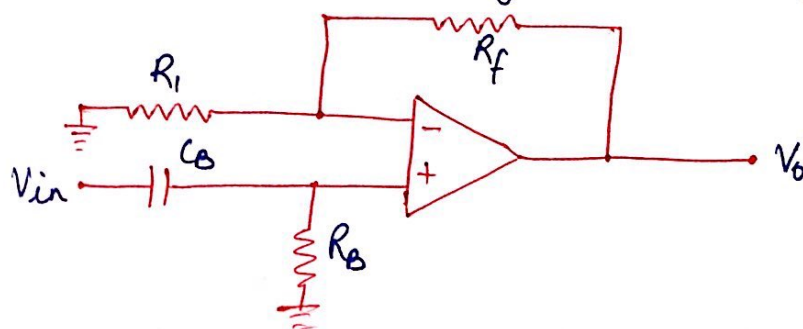


Active LPF Circuit

Diagram

(2) ACTIVE HIGH PASS FILTER

If an active filter passes only high frequency components and blocks all other low frequency components, then it is called as an active high pass filter.

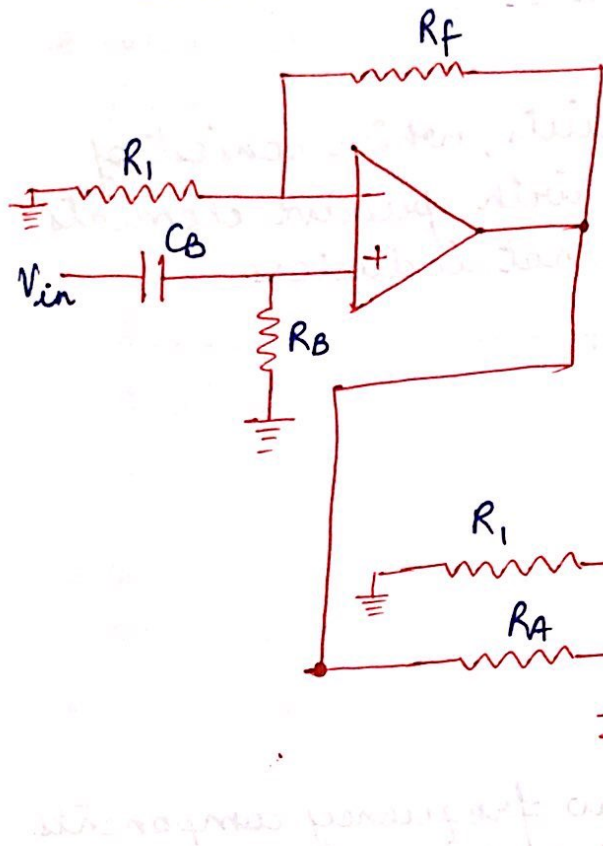


Active HPF Circuit

Diagram

(3) ACTIVE BAND PASS FILTER

- * If an active filter passes only one band of frequency then it is called an active band pass filter.

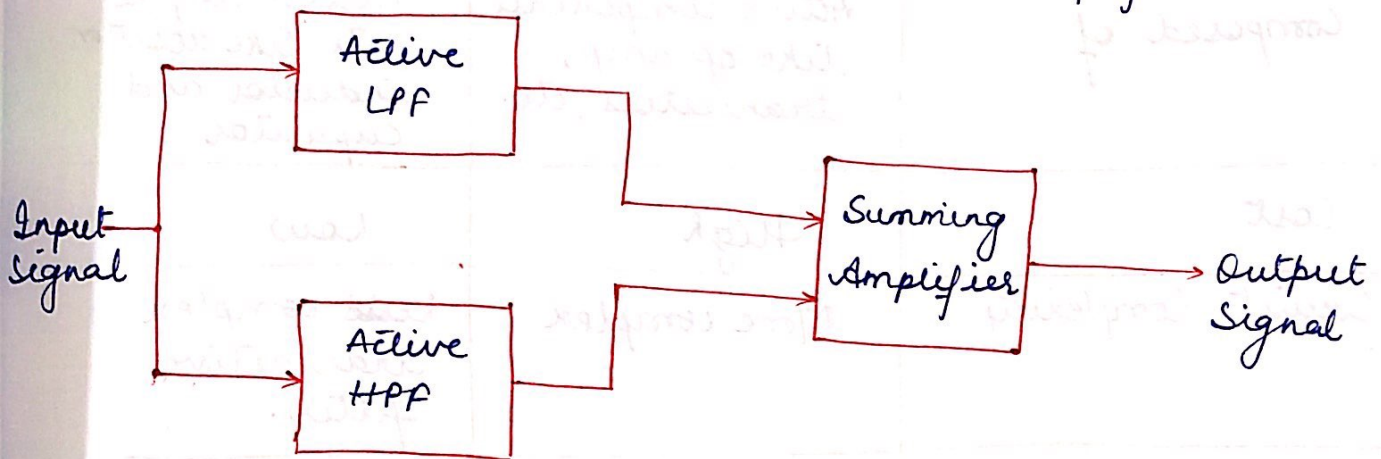


Active Band Pass filter
Circuit Diagram

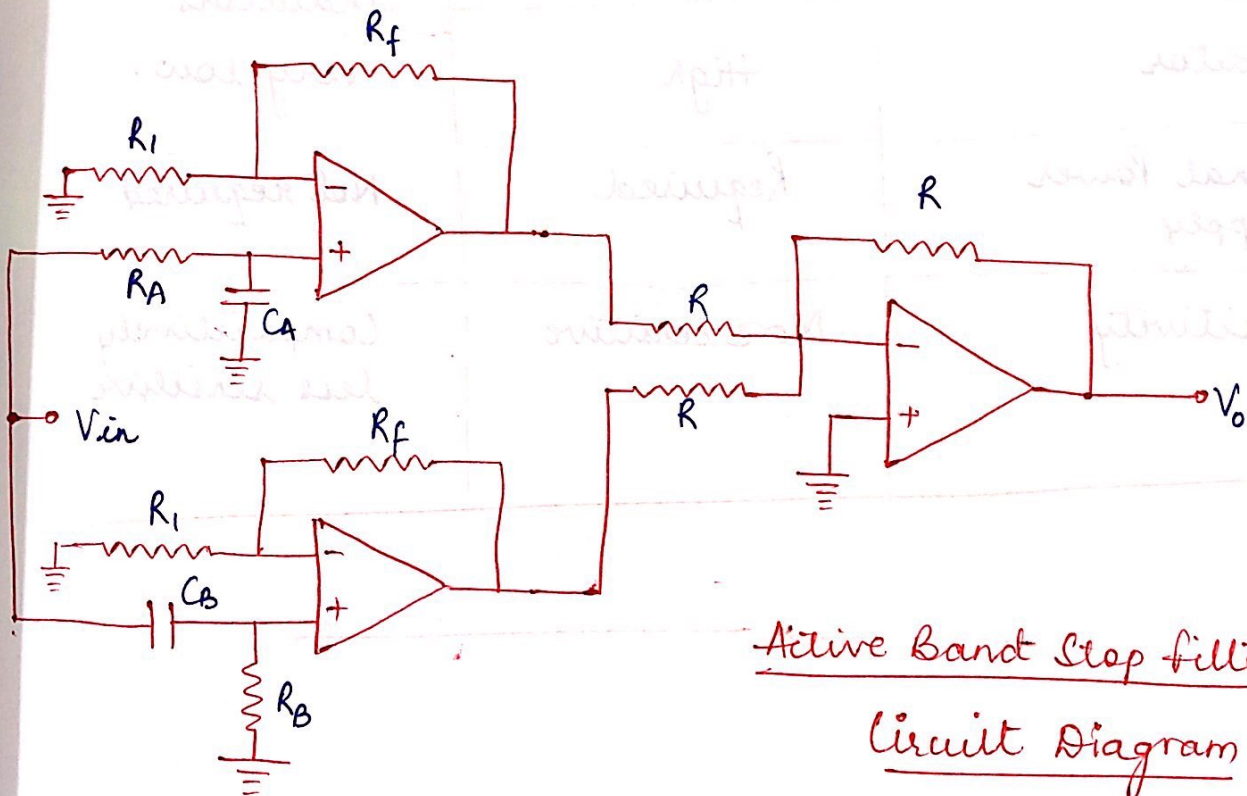
- * There are two parts in the above circuit diagram—
 - The 1st part is an active high pass filter.
 - The 2nd part is an active low pass filter.
- * The output of the active HPF is applied as an input of the active LPF.
- * The active HPF, which is present at the first stage, allows the frequencies that are greater than the lower cut-off frequency, f_L of the active band pass filter.
- * Similarly, the active LPF allows the frequencies that are lower than the higher cut-off frequency f_H of the active band pass filter.

(4) ACTIVE BAND STOP FILTER

If an active filter rejects a particular band of frequencies, then it is called active band stop filter.



Block Diagram of an Active Band Stop filter



Active Band Stop filter
Circuit Diagram

ACTIVE FILTER v/s PASSIVE FILTER

BASIS FOR COMPARISON	ACTIVE FILTER	PASSIVE FILTER
Composed of	Active components like op-amp, transistors, etc.	Passive components like resistor, inductor and capacitor.
Cost	High	Low
Circuit Complexity	More complex	Less complex than active filter.
Weight	Low	Comparitively bulkier due to inductors
Q-factor	High	Very low.
External Power Supply	Required	Not required
Sensitivity	More sensitive	Comparitively less sensitive